Errors Due to Wheatstone Bridge Nonlinearity

1.0 Introduction

Commercial static strain indicators and signal conditioners vary considerably in their circuit details; and, although most of them are based upon some form of the Wheatstone bridge circuit, the bridge circuit is employed in differing ways in different instruments. Because of the many variations in instrument design, a completely general treatment of instrument nonlinearities is not practicable within the scope of this publication. There is, however, a large class of static strain indicators and signal conditioners with a more-or-less characteristic circuit arrangement (employing the "unbalanced" Wheatstone bridge), and displaying a characteristic nonlinearity. This Tech Note has been prepared to provide a simple means for determining the magnitudes of the nonlinearity errors and for making corrections when necessary. Note that the error and correction relationships given here apply only to instruments having the characteristics defined in Section 2.0. For other instruments, the nonlinearity errors, if they exist, will have to be determined by direct calibration or from manufacturers' specifications.

The nonlinearity error occurs because, when strain measurements are made with an "unbalanced" Wheatstone bridge circuit (as described in Section 2.0), there are certain conditions under which the output of the bridge circuit is a nonlinear function of the resistance change(s) producing that output. The error due to the nonlinearity, when present, is ordinarily small, and can usually be ignored when measuring elastic strains in metals. However, the percentage error increases with the magnitude of the strain being measured, and can become quite significant at large strains (for example, the error is about 0.1% at 1000$\mu\epsilon$, 1% at 10 000$\mu\epsilon$, and 10% at 100 000$\mu\epsilon$; or, as a convenient rule of thumb, the error, in percent, is approximately equal to the strain, in percent).

2.0 The "Unbalanced" Wheatstone Bridge Circuit

Most static strain indicators and signal conditioners for use with resistance strain gages use a form of the Wheatstone bridge circuit in which the bridge arms consist of one to four active gages. The classical Wheatstone bridge arrangement has been used for many years for the accurate measurement of a single unknown resistance; and, in such instruments, the bridge is balanced at the time of measurement by adjusting the resistances of the other arms. The bridge circuit found in most strain indicators, on the other hand, is unbalanced by the varying gage resistance(s) at the time of making the measurement, and is therefore commonly referred to as the "unbalanced" Wheatstone bridge.

The output voltage obtained from the "unbalanced" Wheatstone bridge is a function of the amount of unbalance, and is therefore directly related to the strain applied to the strain gage. However, under certain conditions frequently encountered in actual practice, the bridge output voltage is, as noted earlier, a nonlinear function of the resistance change in the bridge arms. When this occurs, the strain readings will be somewhat in error.

Figure 1 shows two of the circuit arrangements most commonly employed in commercial strain indicators and signal conditioners. In circuit (A), the bridge output voltage is amplified and displayed on an indicating instrument, frequently a digital voltmeter. In circuit (B), the bridge output voltage is "nulled" by an equal and opposite voltage injected into the measurement circuit. In both cases, the nonlinearity errors are identical if the amplifiers have high input impedances, and if the power supplies are of the constant-voltage type. Note also that in both circuits the "balance" control is

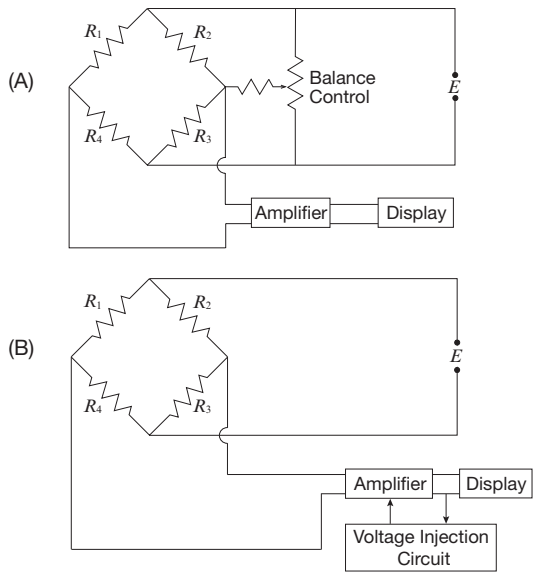


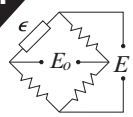
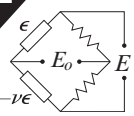
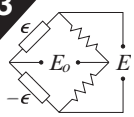
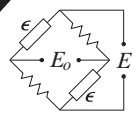
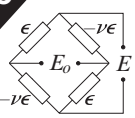
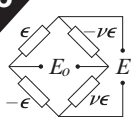
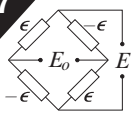
Figure 1. Circuit arrangements for commercial instruments.

TECH NOTE

Errors Due to Wheatstone Bridge Nonlinearity

used only to establish *initial* bridge balance before the gages are strained, and that the balance controls do not form part of the readout circuit. This type of "balance" circuit is normally provided with a limited range so as not to cause problems in resolution and setting-stability; and, therefore does not greatly influence the nonlinearity errors as described in this Tech Note. To permit a rigorous treatment of the errors without introducing other considerations, it is assumed throughout the following discussion that the "balance" circuit is either completely disconnected, or that the control is left at the midpoint of its range. It is also assumed that the bridge arms are nominally resistively symmetrical about an

TABLE 1

Bridge/Strain Arrangement (Note 1)	Description	Bridge Output, E_o/E mV/V (Notes 2, 3)	Nonlinearity, η Where $E_o/E = K\epsilon \times 10^{-3}(1-\eta)$ (Notes 2, 3)	Corrections (Note 3, 4)
1 	Single active gage in uniaxial tension or compression.	$\frac{E_o}{E} = \frac{F\epsilon\times10^{-3}}{4+2F\epsilon\times10^{-6}}$	$K = \frac{F}{4}$ $\eta = \frac{F\epsilon\times10^{-6}}{2+F\epsilon\times10^{-6}}$	$\epsilon = \frac{2\epsilon_i}{2-F\epsilon_i\times10^{-6}}$
2 	Two active gages in uniaxial stress field — one aligned with maximum principal strain, one "Poisson" gage.	$\frac{E_o}{E} = \frac{F\epsilon(1+\nu)\times10^{-3}}{4+2F\epsilon(1-\nu)\times10^{-6}}$	$K = \frac{F(1+\nu)}{4}$ $\eta = \frac{F\epsilon(1-\nu)\times10^{-6}}{2+F\epsilon(1-\nu)\times10^{-6}}$	$\epsilon = \frac{2\epsilon_i}{2(1+\nu)-F\epsilon_i(1-\nu)\times10^{-6}}$
3 	Two active gages with equal and opposite strains — typical of bending-beam arrangement.	$\frac{E_o}{E} = \frac{F\epsilon}{2}\times10^{-3}$	$K = \frac{F}{2};\ \eta = 0$	$\epsilon = \frac{\epsilon_i}{2}$
4 	Two active gages with equal strains of same sign — used on opposite sides of column with low temperature gradient (bending cancellation, for instance).	$\frac{E_o}{E} = \frac{F\epsilon\times10^{-3}}{2+F\epsilon\times10^{-6}}$	$K = \frac{F}{2}$ $\eta = \frac{F\epsilon\times10^{-6}}{2+F\epsilon\times10^{-6}}$	$\epsilon = \frac{2\epsilon_i}{4-F\epsilon_i\times10^{-6}}$
5 	Four active gages in unaxial stress field two aligned with maximum principal strain, two "Poisson" gages (column).	$\frac{E_o}{E} = \frac{F\epsilon(1+\nu)\times10^{-3}}{2+F\epsilon(1-\nu)\times10^{-6}}$	$K = \frac{F(1+\nu)}{2}$ $\eta = \frac{F\epsilon(1-\nu)\times10^{-6}}{2+F\epsilon(1-\nu)\times10^{-6}}$	$\epsilon = \frac{2\epsilon_i}{4(1+\nu)-F\epsilon_i(1-\nu)\times10^{-6}}$
6 	Four active gages in uniaxial stress field — two aligned with maximum principal strain, two "Poisson" gages (beam).	$\frac{E_o}{E} = \frac{F\epsilon(1+\nu)\times10^{-3}}{2}$	$K = \frac{F(1+\nu)}{2};\ \eta = 0$	$\epsilon = \frac{\epsilon_i}{2(1+\nu)}$
7 	Four active gages with pairs subjected to equal and opposite strains (beam in bending or shaft in torsion).	$\frac{E_o}{E} = F\epsilon\times10^{-3}$	$K = F;\ \eta = 0$	$\epsilon = \frac{\epsilon_i}{4}$

Notes:
1. $\langle {}^{1\uparrow 2}_{4\downarrow 3} \rangle$ $(R_1/R_4)_{nom} = 1$; $(R_2/R_3)_{nom} = 1$ when two or less active arms are used.
2. Constant voltage power supply is assumed.
3. ϵ and ϵ_i (strains) are expressed in microstrain units (in/in x 10^6) where ϵ_i is the strain indicated by your instrument and ϵ is the actual strain under a single active gage.
4. Expressions in this column correct for Wheatstone bridge nonlinearity (if present) and for the number of active gages in the circuit.

TECH NOTE

Errors Due to Wheatstone Bridge Nonlinearity

axis joining the output corners of the bridge; i.e., that:

$$(R_1/R_4)_{nom} = 1 = (R_2/R_3)_{nom}$$

As a result of the circuit arrangements described above, obtaining a reading from the instrument (whether or not the process involves nulling a meter) has no effect on the state of resistive balance within the Wheatstone bridge circuit. Even if the Wheatstone bridge is initially balanced resistively so that $R_1/R_4 = R_2/R_3$, this will no longer be true, in general, when one or more of the strain gages in the bridge arms are strained. Consequently, the Wheatstone bridge is ordinarily operated in a resistively unbalanced state. In this mode of operation, resistance changes in the bridge arms may cause changes in the currents through the arms, depending upon the signs and magnitudes of the resistance changes in all four arms. When current changes occur, the voltage output of the bridge is not proportional to the resistance changes, and thus the output is nonlinear with strain, and the instrument indication is in error.

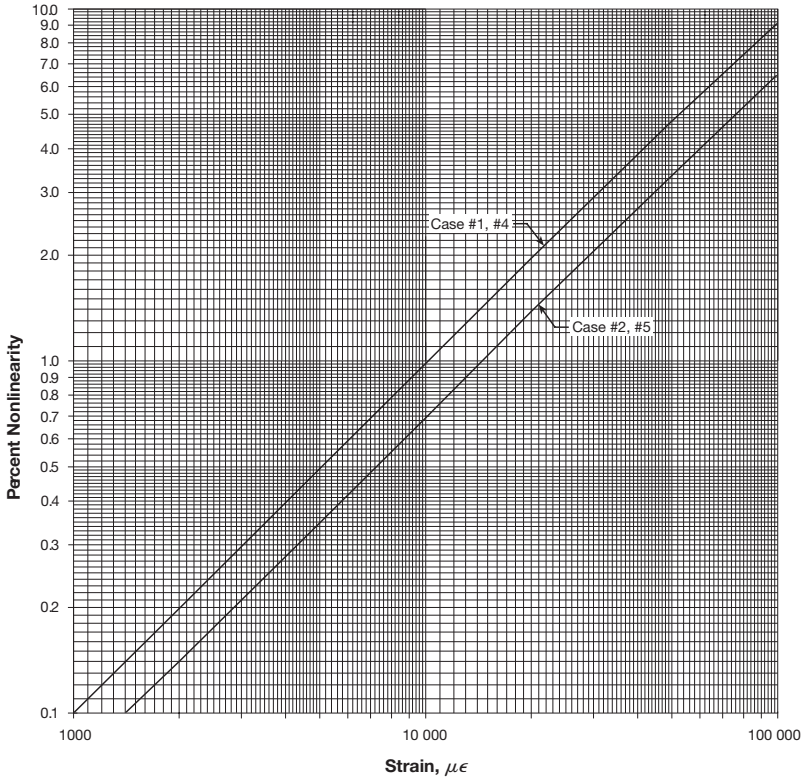


Figure 2. Nonlinearity errors for tensile strain in bridge circuits.

3.0 Nonlinearities and Corrections

For the class of instruments described in Section 2.0, Table 1 illustrates the nonlinearities to be expected, and includes correction relationships. The table gives the nondimensional output voltage (E_o/E) as a function of the applied strain (and gage factor) for a variety of commonly encountered strain states and different arrangements of gages on the structural member and within the Wheatstone bridge. It can readily be seen that the output expressions for cases 1, 2, 4, and 5 are intrinsically nonlinear, while those for cases 3, 6, and 7 are linear. Examination of the column of bridge/strain arrangements demonstrates that only when the resistance changes due to strain are such that the currents through the bridge arms remain constant — that is, $\Delta R_1/R_1 + \Delta R_4/R_4 = 0$ and $\Delta R_2/R_2 + \Delta R_3/R_3 = 0$ — is the output a linear function of the strain.

For each of the cases in Table 1, the nondimensional circuit output can also be expressed in the following form:

$$E_o/E = K\epsilon \times 10^{-3}(1-\eta), \text{ mV/V}$$

In this relationship, K is a constant, determined by the gage factor of the strain gage(s) and the number of active arms in the bridge circuit; and η (when not zero) represents the nonlinearity caused by changes in the currents through the bridge arms. One hundred times the fraction $\eta/1$ is then the percentage nonlinearity in the circuit output.

The nonlinearity column in Table 1 gives the mathematical expression for calculating η as a function of the applied strain and other relevant parameters. It can be noticed from the table that on a percentage basis the nonlinearity magnitudes are identical for cases 1 and 4, and for 2 and 5, although the circuit outputs differ. For convenience in quickly judging nonlinearity magnitudes, the relationships in cases 1, 2, 4, and 5 are plotted in Figure 2, assuming positive (tensile) strains, a gage factor of 2.0, and Poisson's ratio (where involved) of 0.30. The nonlinearity for compressive strain is opposite in sign and somewhat different in magnitude, but can always be calculated from the relationships given in Table 1.

The last (right most) column in Table 1 provides the relationships for converting the indicated strain, ϵ_i, as registered by a strain indicator or other instrument system to the actual surface strain under a single active gage, ϵ. The expressions in this column correct for Wheatstone bridge nonlinearity (when present) and for the number of active gages in the circuit.

4.0 Numerical Examples

For simplicity in presentation, the following three numerical examples assume, in each case, a quarter-bridge circuit

TECH NOTE

Errors Due to Wheatstone Bridge Nonlinearity

with a single active gage, and a gage factor of 2.0. The procedures employed in the examples are, however, quite general, and apply to all circuit arrangements in Table 1.

As a first example, assume that the quarter-bridge circuit was initially balanced resistively with no load applied to the test piece. Subsequently, the test piece was loaded until the strain indicator registered 15 000$\mu\epsilon$ in tension. From Figure 2 for case 1, the nonlinearity in output is approximately 1.5 percent or about 225$\mu\epsilon$. The actual surface strain for an indicator reading of 15 000$\mu\epsilon$ can be obtained from the relationship in the corrections column of Table 1. Substituting $F = 2.0$ and $\epsilon_i = 15\,000$ into the expression $\epsilon = (2\epsilon_i)/(2 - F\epsilon_i \times 10^{-6})$ gives $\epsilon = 15\,228\mu\epsilon$. If the strain indicator had read –15 000$\mu\epsilon$ for compression, the same procedure would yield a surface strain of –14 778$\mu\epsilon$. As demonstrated by these calculations, Wheatstone bridge nonlinearity causes indicated tensile strains to be too low, and indicated compressive strains too high.

It was assumed in the previous example that the Wheatstone bridge was initially in a state of resistive balance. In the practice of experimental stress analysis with strain gages, this may not always be the case. For instance, during the bonding of a strain gage the resistance of the gage may be altered significantly from the manufactured value by poor installation technique. It may also happen that the gage is strained to the plastic range by assembly or preload stresses before subsequent strain measurements are to be made. The initial resistive unbalance, unless it is known to be insignificant, should be measured and properly accounted for in making nonlinearity corrections. When great enough to warrant consideration, the initial unbalance (expressed in strain units) must be added algebraically to any subsequent observed strains so that the nonlinearity correction is based on the *total* (or *net*) unbalance of the Wheatstone bridge at any stage in the measurement process.

For this example, assume that the strain indicator displays an initial unbalance of –4500$\mu\epsilon$ in the installed gage, with no load applied to the test object. This is an *indicated* unbalance, and therefore includes a small nonlinearity error which will be corrected for in this case to illustrate the procedure. Substituting as before into the correction relationship for case 1 in Table 1, the actual initial unbalance (in strain units) is: $\epsilon = -4480\mu\epsilon$. After taking this reading (but not resistively balancing the Wheatstone bridge arms), the test object is subjected to its specified load. The change in strain indication corresponding to the applied load is –8000$\mu\epsilon$. The total indicated unbalance in the Wheatstone bridge is then –12 500$\mu\epsilon$. The calculated correction for this strain indication (case 1, Table 1) yields –12 346$\mu\epsilon$ for the actual total unbalance. The actual applied strain is thus –12 346 – (–4480) = –7866$\mu\epsilon$.

As a final example, consider a case in which the indicated initial unbalance after installing the strain gage was –2500$\mu\epsilon$. Then the gaged member was installed in a structure with an indicated additional assembly strain of –45 500$\mu\epsilon$. After taking this reading, subsequent loading of the structure produced an indicated strain change of 3000$\mu\epsilon$ in the tension direction. What corrections should be made to determine the actual tensile strain caused by loading the structure?

Prior to loading the structure, the Wheatstone bridge was unbalanced by an indicated –48 000$\mu\epsilon$. Substituting into the correction expression in Table 1 for this case, the actual resistive unbalance prior to loading was –45 802$\mu\epsilon$ in strain units. After loading the structure, the indicated unbalance in the Wheatstone bridge was –48 000 + 3000 = –45 000$\mu\epsilon$. The calculated correction for this indicated strain yields –43 062$\mu\epsilon$. The applied tensile strain due to loading the structure was thus –43 062 – (–45 802) = +2740$\mu\epsilon$. This example demonstrates that even with relatively modest *working* strains the nonlinearity error can be very significant (about 10% in this instance) if the Wheatstone bridge is operated far from its resistive balance point.

5.0 Nonlinearities in Shunt Calibration and Dynamic Strain Measurement

The nonlinearity error described in the preceding sections of this Tech Note should always be kept in mind during the shunt calibration of a strain indicator or signal conditioner. In the conventional practice of shunt calibration, the strain gage is momentarily shunted by a large resistor of a magnitude selected to produce a decreased resistance in the Wheatstone bridge arm corresponding to, and precisely simulating, a predetermined compressive strain in the gage (at a specified gage factor). As an alternative, the internal dummy resistance in the adjacent arm of the bridge circuit can be shunted to simulate a tensile strain in the gage.

During shunt calibration, the strain indicated by the instrument will be the same as that for a strain gage at the same level of strain, if the proper calibration resistor is employed. To be precise, the calibration resistor for simulating a specific tensile strain is slightly different from that for the same level of compressive strain because the nonlinearities are different in tension and compression. When simulating small strains (less than, say, 2000$\mu\epsilon$), the tension/compression difference is negligible, and standard shunt calibration resistors can be used in either mode. For accurate calibration at large strains, resistors specifically intended for tension or compression simulation should be selected. Tech Note TN-514, "Shunt Calibration of Strain Gage Instruments", gives the necessary relationships for calculating the appropriate shunt resistances for both cases.

The usual practice, after shunting the active or dummy gage, is to adjust the gage factor or gain control of the instrument to exactly register the simulated strain level.

Errors Due to Wheatstone Bridge Nonlinearity

Subsequent strain measurements at or near the calibrated strain level do not require correction for nonlinearity. However, measurement at a significantly different level will be somewhat in error due to the different nonlinearity at a different strain level. Tech Note TN-514 also gives relationships for adjusting indicated strains to account for calibration at one level, followed by strain measurement at a different level. Although not treated here, leadwire resistance is another factor to be considered in shunt calibration, and Tech Note TN-514 provides relationships to correct for leadwire effects.

Errors can also arise when the initial state of Wheatstone bridge unbalance is different during shunt calibration than it is when strain measurements are to be made. If this situation exists, it is necessary to measure the initial unbalance and determine the actual simulated strain following the procedures demonstrated in the second and third of the preceding numerical examples.

Whenever dynamic strain measurements are made with a Wheatstone bridge circuit, the bridge is always operated in the unbalanced mode. Therefore, the nonlinearities listed in Table 1 of this Tech Note apply to every such dynamic strain measurement — assuming, again, that the bridge is initially balanced resistively. Under these conditions, the error due to the nonlinearity is ordinarily small at typical working strain levels, as illustrated by Figure 2. However, if the bridge is initially unbalanced, the nonlinearity error can be much greater; and, with large initial unbalances, may result in significantly inaccurate strain indications.

6.0 Summary

The nonlinearity errors occurring in conventional strain gage bridge circuits are normally small enough to ignore when measuring modest strain magnitudes such as those encountered in the elastic range of metals (if the bridge is initially balanced resistively). Large resistive unbalances can, on the other hand, lead to sizable errors in strain indication. The relationships and procedures presented in this Tech Note can be used when necessary to correct for such errors. It also follows that for accurate strain measurements, it is imperative to select strain gages with tightly controlled resistance tolerances, and to minimize resistance shifts during gage bonding by carefully following recommended installation techniques.